# First Year Teacher Trainees' Understanding of Geometry

**Christine Lawrie**
**University of New England**

In the early 80s, Mayberry (1981) developed a diagnostic instrument to be used to assess the van Hiele levels of pre-service teachers. The test was designed to be carried out in an interview situation. The Mayberry study has been replicated under Australian conditions in a written format, testing sixty first year primary-teacher trainees. This paper presents the results of the study, comparing them with the results of the Mayberry students, and relating them to their geometric backgrounds. Responses, in general, show that many of the students who had completed a recognised senior secondary geometry course could not display better than Level 2 understanding.

Being able to assess students' levels of understanding is a part of the ability to instruct students at their van Hiele level of insight. To assist in making such an assessment, there needs to be available a reliable diagnostic instrument. In the early 80s, Mayberry (1981, 1983) in her work with pre-service primary teachers, developed a diagnostic instrument for use in an interview situation. In 1992, a written test based on the Mayberry items was developed at the University of New England, and used in a detailed study of the geometric perceptions of 60 Australian primary-teacher trainees. Analysis of the results by concept and by level led to the identification of strengths as well as inconsistencies in the Mayberry items. Lawrie (1993) reported on four main aspects in Mayberry's work which have the potential to lead to the incorrect assessment of a student's van Hiele level of understanding in geometry. In contrast, several items designed by Mayberry to test for Level 4 understanding, were shown by Lawrie (1994) to be capable of indicating, not only a student's ability in deduction, but also a student's degree of reasoning for any van Hiele level. This paper presents the quantitative results of the Australian study, relates the results to the students' geometric backgrounds, and compares them with the assessments of the American students.

## Background

### *The van Hiele Theory*

In the 1950s, Pierre van Hiele and Dina van Hiele-Geldof completed companion PhDs which evolved from the difficulties they had experienced as teachers of geometry in secondary schools. Whereas Dina van Hiele-Geldof explored the teaching phases necessary in order to assist students to move from one level to the next, Pierre van Hiele's work developed the theory involving five levels of insight. A brief description of the first four van Hiele levels as they relate to 2-D geometric figures is given. These are the levels which are commonly displayed by secondary students, and are the levels involved in this study, in the assessment of the students' responses.

- Level 1 Perception is visual only. A figure is seen as a total entity and as a specific shape. Properties play no explicit part in the recognition of the shape.
- Level 2 The figure is now identified by its geometric properties rather than by its overall shape. However, the properties are seen in isolation.
- Level 3 The significance of the properties is seen. Properties are ordered logically and relationships between the properties are recognised.
- Level 4 Logical reasoning is developed. Geometric proofs are constructed with meaning. Necessary and sufficient conditions are used with understanding.

Van Hiele (1986, pp.5-6) maintains that, contrary to the theories of Piaget, progression from one level to the next is not the result of maturation or natural development. He explains (p.39) "The attainment of the new level cannot be effected by teaching, but still, by a suitable choice of exercises the teacher can create a situation for the pupil favourable to the attainment of the higher level of thinking". Students' understanding of a geometry topic undergoes several changes as they continue their studies, the changes reflecting their progress through the van Hiele levels in a particular order. If one level is not mastered before instruction proceeds to the next level, students may perform only algorithmically on the higher level.

## Design

For the study, the Mayberry interview items were modified to ensure clarity of the written questions. A preliminary study validated the reliability of the written questions. The test was restricted to one hour, and hence the seven topics tested by Mayberry were divided between two papers; Paper 1 tested the concepts square, right triangle, circle and congruency, whilst Paper 2 tested the concepts square, isosceles triangle, parallel lines and similarity. The sample consisted of the students enrolled in Mathematics Education I, the first of two compulsory mathematics education courses in the Bachelor of Teaching degree at the University of New England. The test was administered in the second week of the course, before instruction in the space segment. The students were randomly allocated to a paper. Before the test, the students were instructed both orally and in writing, to present as much detail as possible when giving reasons for their answers. The students were also asked to complete an information sheet detailing their mathematical background.

The evaluation method developed by Mayberry was used to assess the students' responses, thus enhancing the comparability of the two sets of results. This method registers whether a student has given sufficient correct answers at a particular level to reach the criterion set for the level. Results are expressed as a quadruple showing the highest van Hiele level of thinking displayed by a student for each of the four concepts. For example, a quadruple (3,3,3,2) resulting from Paper 1 would indicate that a student displayed Level 3 thinking for the concepts square, right triangle and circle, and Level 2 thinking for the concept congruency. A 10% sample of the students were interviewed to validate the results of the written test. The responses to individual questions and to clusters of questions were further analysed to ascertain whether the patterns Mayberry perceived in her evaluation, also occurred in the Australian responses. Mayberry applied Guttman's scalogram analysis, calculating a coefficient of reproducibility of 0.95. This high value was taken as "an indication that the van Hiele levels do form a scale, and that, therefore, the levels as tested by these questions are hierarchical in nature" (1981, p.71). A Guttman coefficient of 0.98 for the Australian study was considered confirmation that the transformation of the interview questions to a written test paper had not affected the essence of the Mayberry test.

## Results

Every endeavour was taken to replicate Mayberry's evaluation of responses. Her thesis was examined in depth to ascertain her expectations in the responses to the items. However, this was not possible for every item, there being occasions, particularly for the Level 3 and Level 4 items, when insufficient information was to be found in the Mayberry writings. The number of correct responses tabled appeared to exceed Mayberry's statements in the text concerning the number and type of responses given for some items. In this report, to facilitate comparison all results are given as percentages, with the horizontal sums in Tables 1, 2, 4 and 5 being 100%.

The results of the assessment of the students' levels of understanding are summarised below in Table 1, whilst Table 2 shows the comparable results for the Mayberry subjects. Some students failed to identify concepts. Those students are recorded as achieving No Level. The results show that, for both studies, the majority of students were assessed as having no greater than Level 2 understanding, i.e. they were comfortable recognising concepts, and listing the associated properties, but did not understand the relationships between the properties. The tables also show that the most familiar topic for both sets of students appears to be the circle. However, there is no evidence to support the notion that students find the concept of the circle easier to understand than other common 2-dimensional shapes. Rather, the detailed investigation into the Mayberry test items has shown that the better-than-expected results for the circle are attributable to weaknesses in the design of several of the questions (Lawrie 1993, pp.383-385).

Table 1
Highest level reached by the Australian students for each concept
(% of sample)

| Concept | No Level | Level 1 | Level 2 | Level 3 | Level 4 |
|---|---|---|---|---|---|
| Square | 0 | 3 | 84 | 7 | 7 |
| Right Triangle | 3 | 19 | 55 | 19 | 3 |
| Isosceles Triangle | 7 | 27 | 43 | 20 | 3 |
| Circle | 0 | 13 | 19 | 52 | 16 |
| Parallel Lines | 0 | 17 | 80 | 0 | 3 |
| Congruency | 0 | 32 | 35 | 3 | 29 |
| Similarity | 0 | 43 | 40 | 10 | 7 |

Table 2
Highest level reached by the Mayberry students for each concept
(% of sample)

| Concept | No Level | Level 1 | Level 2 | Level 3 | Level 4 |
|---|---|---|---|---|---|
| Square | 0 | 11 | 32 | 26 | 32 |
| Right triangle | 26 | 21 | 21 | 16 | 16 |
| Isosceles triangle | 26 | 16 | 11 | 26 | 21 |
| Circle | 5 | 11 | 16 | 21 | 47 |
| Parallel lines | 26 | 16 | 16 | 37 | 5 |
| Congruency | 0 | 21 | 32 | 21 | 26 |
| Similarity | 5 | 42 | 5 | 21 | 26 |

A most common level of working was considered to be a useful descriptor, particularly if wanting to refer to the overall insight of a student. This most common level was determined for each student by calculating the mean of the four assessments listed in the quadruple. In the example given above, the quadruple (3,3,3,2) has a mean of 2.75. This is rounded to a whole number, resulting in the student being said to have a most common level of working of Level 3. Table 3 shows the percentage of students most commonly working at each van Hiele level. Although three students each failed to recognise one of the concepts, all students showed a most common level of understanding of at least Level 1. This table again indicates that the majority of students lacked understanding of the relationships between properties, and of knowledge of formal proof.

Table 3
Australian Students' most common van Hiele working level

| van Hiele level | Percentage of students |
|---|---|
| 1 | 8 |
| 2 | 69 |
| 3 | 10 |
| 4 | 13 |

Table 4 relates the students' secondary geometric backgrounds to the level of understanding each has most commonly displayed in their responses to the test. Of the Australian students, 64% had completed a senior secondary mathematics course which included a formal or recognised geometry segment (e.g., NSW 2 Unit Mathematics, Queensland Mathematics I), 23.5% had completed a senior secondary mathematics course which did not contain a formal geometry segment (e.g., Mathematics in Society), and 12.5% had not completed any senior secondary mathematics. It is significant that 63% of the students who had completed a course in which the instruction is assumed to be at van Hiele Level 3 and possibly higher, could not display overall understanding of Level 3 knowledge in their responses.

Table 4
Relationship between the Australian students' geometric backgrounds and their most common van Hiele working level
(% of sample)

| Geometric Background | van Hiele Level 1 | van Hiele Level 2 | van Hiele Level 3 | van Hiele Level 4 |
|---|---|---|---|---|
| Senior geometry | 0 | 63 | 14 | 23 |
| Senior maths but no geometry | 8 | 92 | 0 | 0 |
| No senior maths | 43 | 43 | 14 | 0 |

Mathematics is commonly studied in High Schools in the USA (Years 9/10 to Year 12), as separate optional courses, e.g., algebra, calculus, geometry. Mayberry's examination of high school geometry textbooks (1983, p.68) showed that "Level 3 thought appears to be needed to begin the course and that Level 4 thought should be developed during the course." In Australia, most of the mathematics courses offered in senior secondary schools have a composite syllabus, the geometry segment of which generally appears to be designed for Level 3 (and sometimes Level 4) instruction. For example, notes on content of the plane geometry segment of the NSW mathematics syllabus for the 2 Unit and 3 Unit courses (Board of Senior School Studies, 1982, p.14) state that students are expected to have "a knowledge of the various common geometrical figures and their properties" (van Hiele Levels 1 and 2). The content includes the development of the understanding of notions, and of the ability to provide proofs for deductive exercises, i.e., the development of van Hiele Level 3 and early Level 4 competency. In Mayberry's study, 68% of the subjects had taken geometry as a course in High School, and 32% had not. This is similar to the composition of the Australian sample in which 64% had studied senior geometry and 36% had not. For Australian and American students with similar geometric backgrounds, Table 5 compares the highest van Hiele level achieved by each student for each concept.

Table 5
Comparison of highest levels reached by students who have taken senior geometry and those who have not, for Australian and Mayberry students
(% of sample)

| Highest level reached | | No Level | Level 1 | Level 2 | Level 3 | Level 4 |
|---|---|---|---|---|---|---|
| Geometry background | Australian | 1 | 12 | 50 | 19 | 18 |
| | Mayberry | 13 | 16 | 12 | 29 | 30 |
| No geometry background | Australian | 3 | 33 | 55 | 9 | 1 |
| | Mayberry | 12 | 26 | 33 | 14 | 14 |

Of the students who had completed a recognised geometry course in which the content was at van Hiele Level 3 or higher, only 37% of the Australians and 59% of Mayberry's subjects could demonstrate Level 3 or better understanding in their responses. The differences in the success rates between the two countries may be a reflection of the optional nature of the American geometry course, this attracting students more appreciative of geometry. However, the generally low number of students displaying a level of insight which matches the level of instruction in the senior geometry courses may be an indication that many students have learned by rote to manipulate the 'knowledge' of geometry which they possess but do not understand.

Mayberry's examination of individual questions and clusters of questions led her to summarise many of the patterns displayed by students in their responses to individual questions and clusters of questions (1981, p.89). Most of these patterns appeared also in the Australian results. Overall, the analysis of the responses to individual questions and to clusters of questions in this study, indicated that:

1 some students have difficulty in recognising a figure in a non-standard position;
2 properties of sides of figures are more readily identified than properties of angles;
3 some students working at Level 2, property identification, resort to quantifying a figure when tackling a Level 3 problem;
4 other students, unable to work with a generalised figure, draw a particular geometric figure;
5 many students do not have an understanding of class inclusion, for example, do not recognise a square as a rectangle;
6 most students do not understand the difference between necessary and sufficient conditions;
7 on only three occasions did students fail to reach the first level, all occasions occurring with the triangle concept;
8 the majority of students who had completed a senior secondary geometry course in which the content was Level 3 or higher, were not able to demonstrate Level 3 insight in their responses.

## Conclusion

The distribution of the Australian results was similar to that of the Mayberry results. More than half the students from both countries were unable to demonstrate skills for levels higher than Level 2. Low levels of understanding (Levels 1 and 2) were displayed both by students who had completed a recognised senior geometry course, as well as by those who had not. However, a higher percentage of the American students who had completed a senior geometry course were shown as demonstrating ability at Levels 3 and 4. This difference between the results of the students from the two countries needs to be investigated further. First, there needs to be an examination into whether the responses accepted for the Mayberry and the Australian studies were of the same level. As mentioned earlier, every endeavour was taken to replicate Mayberry's expectations in the responses. However, there were occasions with the higher level items, when there was insufficient or confusing information concerning what was an acceptable answer. Investigation also needs to be carried out comparing the content of American and Australian courses in geometry, the time spent teaching these courses, and whether the teaching in the USA is more effective than in Australia, i.e., is the instruction of the majority of the students at their van Hiele level of insight? If this last factor is true, then to what degree is this readiness of the students for Level 3 instruction a result of geometry being offered as an elective course rather than as a compulsory segment of a composite course?

In a paper he wrote in 1959 (cited by Mayberry 1983, p.67), van Hiele stated two implications of his theory:

(a) a student cannot function adequately at a level without having had experiences that enable the student to think intuitively at each preceding level;

(b) if the language of instruction is at a higher level than a student's insight, the student will not understand the instruction.

The results of both studies indicate that many of the students attempting senior geometry may not have been at an appropriate level to understand the instruction. This appears to have led to many senior secondary students learning by rote to manipulate the 'knowledge' of geometry which they possess but do not understand and of which they have not seen the origin. They have been offered a system of relations ready-made, and have not learned to establish the connections between the relations, and do not know how to apply what they have learned.

In Australia, there is much emphasis on the final results for secondary schooling, this often serving as an admission measure for tertiary education. Emphasis on the scoring outcome can have an adverse affect on the amount of time spent in the classroom which is directed towards the discovery of knowledge. This can result in the giving of subject matter or tasks which surpass the comprehension of the students. The teachers sometimes quieten their consciences by pretending that the final score is what is important. Van Hiele (1957, p.241) noted that "both examinations and test-papers tend to push the pupil towards algorithmical insight instead of leading him on towards the far more valuable higher forms of insight."

If students are to be instructed at their van Hiele level of insight, the nature of mathematics as described in the general principles which underlie, for example, the NSW syllabuses, needs to be kept in mind:

Mathematics is:

- a search for patterns and relationships
- a way of thinking
- a powerful, precise and concise means of communication
- a creative activity. Therefore it may involve invention, intuition and discovery.

NSW Department of Education, 1989, p.2

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